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# Common fixed point of a power graphic contraction pair in partial metric spaces endowed with a graph

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## Abstract

In this paper, we initiate a study of fixed point results in the setup of partial metric spaces endowed with a graph. The concept of a power graphic contraction pair of two mappings is introduced. Common fixed point results for such maps without appealing to any form of commutativity conditions defined on a partial metric space endowed with a directed graph are obtained. These results unify, generalize and complement various known comparable results from the current literature.

**MSC:** 47H10; 54H25; 54E50**Keywords:** partial metric space; common fixed point; directed graph; power graphic contraction pair

## 1 Introduction and preliminaries

Consistent with Jachymski [1], let  $X$  be a nonempty set and  $d$  be a metric on  $X$ . A set  $\{(x, x) : x \in X\}$  is called a diagonal of  $X \times X$  and is denoted by  $\Delta$ . Let  $G$  be a directed graph such that the set  $V(G)$  of its vertices coincides with  $X$  and  $E(G)$  is the set of the edges of the graph with  $\Delta \subseteq E(G)$ . Also assume that the graph  $G$  has no parallel edges. One can identify a graph  $G$  with the pair  $(V(G), E(G))$ . Throughout this paper, the letters  $\mathbb{R}$ ,  $\mathbb{R}^+$ ,  $\omega$  and  $\mathbb{N}$  will denote the set of real numbers, the set of nonnegative real numbers, the set of nonnegative integers and the set of positive integers, respectively.

**Definition 1.1** [1] A mapping  $f : X \rightarrow X$  is called a Banach  $G$ -contraction or simply  $G$ -contraction if

- (a<sub>1</sub>) for each  $x, y \in X$  with  $(x, y) \in E(G)$ , we have  $(f(x), f(y)) \in E(G)$ ,
- (a<sub>2</sub>) there exists  $\alpha \in (0, 1)$  such that for all  $x, y \in X$  with  $(x, y) \in E(G)$  implies that  $d(f(x), f(y)) \leq \alpha d(x, y)$ .

Let  $X^f := \{x \in X : (x, f(x)) \in E(G) \text{ or } (f(x), x) \in E(G)\}$ .

Recall that if  $f : X \rightarrow X$ , then a set  $\{x \in X : x = f(x)\}$  of all fixed points of  $f$  is denoted by  $F(f)$ . A self-mapping  $f$  on  $X$  is said to be

- (1) a Picard operator if  $F(f) = \{x^*\}$  and  $f^n(x) \rightarrow x^*$  as  $n \rightarrow \infty$  for all  $x \in X$ ;
- (2) a weakly Picard operator if  $F(f) \neq \emptyset$  and for each  $x \in X$ , we have  $f^n(x) \rightarrow x^* \in F(f)$  as  $n \rightarrow \infty$ ;

(3) orbitally continuous if for all  $x, a \in X$ , we have

$$\lim_{k \rightarrow \infty} f^{nk}(x) = a \quad \text{implies} \quad \lim_{i \rightarrow \infty} f(f^{ni}(x)) = f(a).$$

The following definition is due to Chifu and Petrusel [2].

**Definition 1.2** An operator  $f : X \rightarrow X$  is called a Banach  $G$ -graphic contraction if

- (b<sub>1</sub>) for each  $x, y \in X$  with  $(x, y) \in E(G)$ , we have  $(f(x), f(y)) \in E(G)$ ,
- (b<sub>2</sub>) there exists  $\alpha \in [0, 1)$  such that

$$d(f(x), f^2(x)) \leq \alpha d(x, f(x)) \quad \text{for all } x \in X^f.$$

If  $x$  and  $y$  are vertices of  $G$ , then a path in  $G$  from  $x$  to  $y$  of length  $k \in \mathbb{N}$  is a finite sequence  $\{x_n\}$ ,  $n \in \{0, 1, 2, \dots, k\}$  of vertices such that  $x_0 = x$ ,  $x_k = y$  and  $(x_{i-1}, x_i) \in E(G)$  for  $i \in \{1, 2, \dots, k\}$ .

Notice that a graph  $G$  is connected if there is a path between any two vertices and it is weakly connected if  $\tilde{G}$  is connected, where  $\tilde{G}$  denotes the undirected graph obtained from  $G$  by ignoring the direction of edges. Denote by  $G^{-1}$  the graph obtained from  $G$  by reversing the direction of edges. Thus,

$$E(G^{-1}) = \{(x, y) \in X \times X : (y, x) \in E(G)\}.$$

Since it is more convenient to treat  $\tilde{G}$  as a directed graph for which the set of its edges is symmetric, under this convention, we have that

$$E(\tilde{G}) = E(G) \cup E(G^{-1}).$$

If  $G$  is such that  $E(G)$  is symmetric, then for  $x \in V(G)$ , the symbol  $[x]_G$  denotes the equivalence class of the relation  $R$  defined on  $V(G)$  by the rule:

$yRz$  if there is a path in  $G$  from  $y$  to  $z$ .

A graph  $G$  is said to satisfy the property (A) (see also [2]) if for any sequence  $\{x_n\}$  in  $V(G)$  with  $x_n \rightarrow x$  as  $n \rightarrow \infty$  and  $(x_n, x_{n+1}) \in E(G)$  for  $n \in \mathbb{N}$  implies that  $(x_n, x) \in E(G)$ .

Jachymski [1] obtained the following fixed point result for a mapping satisfying the Banach  $G$ -contraction condition in metric spaces endowed with a graph.

**Theorem 1.3** [1] *Let  $(X, d)$  be a complete metric space and  $G$  be a directed graph and let the triple  $(X, d, G)$  have a property (A). Let  $f : X \rightarrow X$  be a  $G$ -contraction. Then the following statements hold:*

1.  $F_f \neq \emptyset$  if and only if  $X_f \neq \emptyset$ ;
2. if  $X_f \neq \emptyset$  and  $G$  is weakly connected, then  $f$  is a Picard operator;
3. for any  $x \in X_f$  we have that  $f|_{[x]_{\tilde{G}}}$  is a Picard operator;
4. if  $f \subseteq E(G)$ , then  $f$  is a weakly Picard operator.

Gwóźdz-Lukawska and Jachymski [3] developed the Hutchinson-Barnsley theory for finite families of mappings on a metric space endowed with a directed graph. Bojor [4] obtained a fixed point of a  $\varphi$ -contraction in metric spaces endowed with a graph (see also [5]). For more results in this direction, we refer to [2, 6, 7].

On the other hand, Mathews [8] introduced the concept of a partial metric to obtain appropriate mathematical models in the theory of computation and, in particular, to give a modified version of the Banach contraction principle more suitable in this context. For examples, related definitions and work carried out in this direction, we refer to [9–19] and the references mentioned therein. Abbas *et al.* [20] proved some common fixed points in partially ordered metric spaces (see also [21]). Gu and He [22] proved some common fixed point results for self-maps with twice power type  $\Phi$ -contractive condition. Recently, Gu and Zhang [23] obtained some common fixed point theorems for six self-mappings with twice power type contraction condition.

Throughout this paper, we assume that a nonempty set  $X = V(G)$  is equipped with a partial metric  $p$ , a directed graph  $G$  has no parallel edge and  $G$  is a weighted graph in the sense that each vertex  $x$  is assigned the weight  $p(x, x)$  and each edge  $(x, y)$  is assigned the weight  $p(x, y)$ . As  $p$  is a partial metric on  $X$ , the weight assigned to each vertex  $x$  need not be zero and whenever a zero weight is assigned to some edge  $(x, y)$ , it reduces to a loop  $(x, x)$ .

Also, the subset  $W(G)$  of  $V(G)$  is said to be complete if for every  $x, y \in W(G)$ , we have  $(x, y) \in E(G)$ .

**Definition 1.4** Self-mappings  $f$  and  $g$  on  $X$  are said to form a power graphic contraction pair if

- (a) for every vertex  $v$  in  $G$ ,  $(v, fv)$  and  $(v, gv) \in E(G)$ ,
- (b) there exists  $\phi : \mathbb{R}^+ \rightarrow \mathbb{R}^+$  an upper semi-continuous and nondecreasing function with  $\phi(t) < t$  for each  $t > 0$  such that

$$p^\delta(fx, gy) \leq \phi(p^\alpha(x, y)p^\beta(x, fx)p^\gamma(y, gy)) \quad (1.1)$$

for all  $(x, y) \in E(G)$  holds, where  $\alpha, \beta, \gamma \geq 0$  with  $\delta = \alpha + \beta + \gamma \in (0, \infty)$ .

If we take  $f = g$ , then the mapping  $f$  is called a power graphic contraction.

The aim of this paper is to investigate the existence of common fixed points of a power graphic contraction pair in the framework of complete partial metric spaces endowed with a graph. Our results extend and strengthen various known results [8, 12, 13, 24].

## 2 Common fixed point results

We start with the following result.

**Theorem 2.1** Let  $(X, p)$  be a complete partial metric space endowed with a directed graph  $G$ . If  $f, g : X \rightarrow X$  form a power graphic contraction pair, then the following hold:

- (i)  $F(f) \neq \emptyset$  or  $F(g) \neq \emptyset$  if and only if  $F(f) \cap F(g) \neq \emptyset$ .
- (ii) If  $u \in F(f) \cap F(g)$ , then the weight assigned to the vertex  $u$  is 0.
- (iii)  $F(f) \cap F(g) \neq \emptyset$  provided that  $G$  satisfies the property (A).
- (iv)  $F(f) \cap F(g)$  is complete if and only if  $F(f) \cap F(g)$  is a singleton.

*Proof* To prove (i), let  $u \in F(f)$ . By the given assumption,  $(u, gu) \in E(G)$ . Assume that we assign a non-zero weight to the edge  $(u, gu)$ . As  $(u, u) \in E(G)$  and  $f$  and  $g$  form a power graphic contraction, we have

$$\begin{aligned} p^\delta(u, gu) &= p^\delta(fu, gu) \\ &\leq \phi(p^\alpha(u, u)p^\beta(u, fu)p^\gamma(u, gu)) \\ &= \phi(p^{\alpha+\beta}(u, u)p^\gamma(u, gu)) \\ &\leq \phi(p^{\alpha+\beta}(u, gu)p^\gamma(u, gu)) \\ &= \phi(p^\delta(u, gu)) \\ &< p^\delta(u, gu), \end{aligned}$$

a contradiction. Hence, the weight assigned to the edge  $(u, gu)$  is zero and so  $u = gu$ . Therefore,  $u \in F(f) \cap F(g) \neq \emptyset$ . Similarly, if  $u \in F(g)$ , then we have  $u \in F(f)$ . The converse is straightforward.

Now, let  $u \in F(f) \cap F(g)$ . Assume that the weight assigned to the vertex  $u$  is not zero, then from (1.1), we have

$$\begin{aligned} p^\delta(u, u) &= p^\delta(fu, gu) \\ &\leq \phi(p^\alpha(u, u)p^\beta(u, fu)p^\gamma(u, gu)) \\ &= \phi(p^{\alpha+\beta+\gamma}(u, u)) \\ &= \phi(p^\delta(u, u)) \\ &< p^\delta(u, u), \end{aligned}$$

a contradiction. Hence, (ii) is proved.

To prove (iii), we will first show that there exists a sequence  $\{x_n\}$  in  $X$  with  $fx_{2n} = x_{2n+1}$  and  $gx_{2n+1} = x_{2n+2}$  for all  $n \in \mathbb{N}$  with  $(x_n, x_{n+1}) \in E(G)$ , and  $\lim_{n \rightarrow \infty} p(x_n, x_{n+1}) = 0$ .

Let  $x_0$  be an arbitrary point of  $X$ . If  $fx_0 = x_0$ , then the proof is finished, so we assume that  $fx_0 \neq x_0$ . As  $(x_0, fx_0) \in E(G)$ , so  $(x_0, x_1) \in E(G)$ . Also,  $(x_1, gx_1) \in E(G)$  gives  $(x_1, x_2) \in E(G)$ . Continuing this way, we define a sequence  $\{x_n\}$  in  $X$  such that  $(x_n, x_{n+1}) \in E(G)$  with  $fx_{2n} = x_{2n+1}$  and  $gx_{2n+1} = x_{2n+2}$  for  $n \in \mathbb{N}$ .

We may assume that the weight assigned to each edge  $(x_{2n}, x_{2n+1})$  is non-zero for all  $n \in \mathbb{N}$ . If not, then  $x_{2k} = x_{2k+1}$  for some  $k$ , so  $fx_{2k} = x_{2k+1} = x_{2k}$ , and thus  $x_{2k} \in F(f)$ . Hence,  $x_{2k} \in F(f) \cap F(g)$  by (i). Now, since  $(x_{2n}, x_{2n+1}) \in E(G)$ , so from (1.1), we have

$$\begin{aligned} p^\delta(x_{2n+1}, x_{2n+2}) &= p^\delta(fx_{2n}, gx_{2n+1}) \\ &\leq \phi(p^\alpha(x_{2n}, x_{2n+1})p^\beta(x_{2n}, fx_{2n})p^\gamma(x_{2n+1}, gx_{2n+1})) \\ &= \phi(p^\alpha(x_{2n}, x_{2n+1})p^\beta(x_{2n}, x_{2n+1})p^\gamma(x_{2n+1}, x_{2n+2})) \\ &= \phi(p^{\alpha+\beta}(x_{2n}, x_{2n+1})p^\gamma(x_{2n+1}, x_{2n+2})) \\ &< p^{\alpha+\beta}(x_{2n}, x_{2n+1})p^\gamma(x_{2n+1}, x_{2n+2}), \end{aligned}$$

which implies that

$$p^{\alpha+\beta}(x_{2n+1}, x_{2n+2}) < p^{\alpha+\beta}(x_{2n}, x_{2n+1}),$$

a contradiction if  $\alpha + \beta = 0$ . So, take  $\alpha + \beta > 0$ , and we have

$$p(x_{2n+1}, x_{2n+2}) < p(x_{2n}, x_{2n+1})$$

for all  $n \in \mathbb{N}$ . Again from (1.1), we have

$$\begin{aligned} p^\delta(x_{2n+2}, x_{2n+3}) &= p^\delta(gx_{2n+1}, fx_{2n+2}) \\ &= p^\delta(fx_{2n+2}, gx_{2n+1}) \\ &\leq \phi(p^\alpha(x_{2n+2}, x_{2n+1})p^\beta(x_{2n+2}, fx_{2n+2})p^\gamma(x_{2n+1}, gx_{2n+1})) \\ &= \phi(p^\alpha(x_{2n+1}, x_{2n+2})p^\beta(x_{2n+2}, x_{2n+3})p^\gamma(x_{2n+1}, x_{2n+2})) \\ &= \phi(p^{\alpha+\gamma}(x_{2n+1}, x_{2n+2})p^\beta(x_{2n+2}, x_{2n+3})) \\ &< p^{\alpha+\gamma}(x_{2n+1}, x_{2n+2})p^\beta(x_{2n+2}, x_{2n+3}), \end{aligned}$$

which implies that

$$p^{\alpha+\gamma}(x_{2n+2}, x_{2n+3}) < p^{\alpha+\gamma}(x_{2n+1}, x_{2n+2}).$$

We arrive at a contradiction in case  $\alpha + \gamma = 0$ . Therefore, we must take  $\alpha + \gamma > 0$ ; consequently, we have

$$p(x_{2n+2}, x_{2n+3}) < p(x_{2n+1}, x_{2n+2})$$

for all  $n \in \mathbb{N}$ . Hence,

$$p^\delta(x_n, x_{n+1}) \leq \phi(p^\delta(x_{n-1}, x_n)) < p^\delta(x_{n-1}, x_n) \quad (2.1)$$

for all  $n \in \mathbb{N}$ . Therefore, the decreasing sequence of positive real numbers  $\{p^\delta(x_n, x_{n+1})\}$  converges to some  $c \geq 0$ . If we assume that  $c > 0$ , then from (2.1) we deduce that

$$0 < c \leq \limsup_{n \rightarrow \infty} \phi(p^\delta(x_{n-1}, x_n)) \leq \phi(c) < c,$$

a contradiction. So,  $c = 0$ , that is,  $\lim_{n \rightarrow \infty} p^\delta(x_n, x_{n+1}) = 0$  and so we have  $\lim_{n \rightarrow \infty} p(x_n, x_{n+1}) = 0$ . Also,

$$p^\delta(x_n, x_{n+1}) \leq \phi(p^\delta(x_{n-1}, x_n)) \leq \cdots \leq \phi^n(p^\delta(x_0, x_1)). \quad (2.2)$$

Now, for  $m, n \in \mathbb{N}$  with  $m > n$ ,

$$\begin{aligned} p^\delta(x_n, x_m) &\leq p^\delta(x_n, x_{n+1}) + p^\delta(x_{n+1}, x_{n+2}) + \cdots + p^\delta(x_{m-1}, x_m) \\ &\quad - p^\delta(x_{n+1}, x_{n+1}) - p^\delta(x_{n+2}, x_{n+2}) - \cdots - p^\delta(x_{m-1}, x_{m-1}) \\ &\leq \phi^n(p^\delta(x_0, x_1)) + \phi^{n+1}(p^\delta(x_0, x_1)) + \cdots + \phi^{m-1}(p^\delta(x_0, x_1)) \end{aligned}$$

implies that  $p^\delta(x_n, x_m)$  converges to 0 as  $n, m \rightarrow \infty$ . That is,  $\lim_{n, m \rightarrow \infty} p(x_n, x_m) = 0$ . Since  $(X, p)$  is complete, following similar arguments to those given in Theorem 2.1 of [9], there exists a  $u \in X$  such that  $\lim_{n, m \rightarrow \infty} p(x_n, x_m) = \lim_{n \rightarrow \infty} p(x_n, u) = p(u, u) = 0$ . By the given hypothesis,  $(x_{2n}, u) \in E(G)$  for all  $n \in \mathbb{N}$ . We claim that the weight assigned to the edge  $(u, gu)$  is zero. If not, then as  $f$  and  $g$  form a power graphic contraction, so we have

$$\begin{aligned} p^\delta(x_{2n+1}, u) &= p^\delta(fx_{2n}, gu) \\ &\leq \phi(p^\alpha(x_{2n}, u)p^\beta(x_{2n}, fx_{2n})p^\gamma(u, gu)) \\ &= \phi(p^\alpha(x_{2n}, u)p^\beta(x_{2n}, x_{2n+1})p^\gamma(u, gu)). \end{aligned} \quad (2.3)$$

We deduce, by taking upper limit as  $n \rightarrow \infty$  in (2.3), that

$$\begin{aligned} p^\delta(u, gu) &\leq \limsup_{n \rightarrow \infty} \phi(p^\alpha(x_{2n}, u)p^\beta(x_{2n}, x_{2n+1})p^\gamma(u, gu)) \\ &\leq \phi(p^\alpha(u, u)p^\beta(u, u)p^\gamma(u, gu)) \\ &\leq \phi(p^{\alpha+\beta+\gamma}(u, gu)) \\ &< p^\delta(u, gu), \end{aligned}$$

a contradiction. Hence,  $u = gu$  and  $u \in F(f) \cap F(g)$  by (i).

Finally, to prove (iv), suppose the set  $F(f) \cap F(g)$  is complete. We are to show that  $F(f) \cap F(g)$  is a singleton. Assume on the contrary that there exist  $u$  and  $v$  such that  $u, v \in F(f) \cap F(g)$  but  $u \neq v$ . As  $(u, v) \in E(G)$  and  $f$  and  $g$  form a power graphic contraction, so

$$\begin{aligned} 0 &< p^\delta(u, v) = p^\delta(fu, fv) \\ &\leq \phi(p^\alpha(u, v)p^\beta(u, fu)p^\gamma(v, gv)) \\ &= \phi(p^\alpha(u, v)p^\beta(u, u)p^\gamma(v, v)) \\ &\leq \phi(p^\delta(u, v)), \end{aligned}$$

a contradiction. Hence,  $u = v$ . Conversely, if  $F(f) \cap F(g)$  is a singleton, then it follows that  $F(f) \cap F(g)$  is complete.  $\square$

**Corollary 2.2** *Let  $(X, p)$  be a complete partial metric space endowed with a directed graph  $G$ . If we replace (1.1) by*

$$p^\delta(f^s x, g^t y) \leq \phi(p^\alpha(x, y)p^\beta(x, f^s x)p^\gamma(y, g^t y)), \quad (2.4)$$

where  $\alpha, \beta, \gamma \geq 0$  with  $\delta = \alpha + \beta + \gamma \in (0, \infty)$  and  $s, t \in \mathbb{N}$ , then the conclusions obtained in Theorem 2.1 remain true.

*Proof* It follows from Theorem 2.1, that  $F(f^s) \cap F(g^t)$  is a singleton provided that  $F(f^s) \cap F(g^t)$  is complete. Let  $F(f^s) \cap F(g^t) = \{w\}$ , then we have  $f(w) = f(f^s(w)) = f^{s+1}(w) = f^s(f(w))$ , and  $g(w) = g(g^t(w)) = g^{t+1}(w) = g^t(g(w))$  implies that  $fw$  and  $gw$  are also in  $F(f^s) \cap F(g^t)$ . Since  $F(f^s) \cap F(g^t)$  is a singleton, we deduce that  $w = fw = gw$ . Hence,  $F(f) \cap F(g)$  is a singleton.  $\square$

The following remark shows that different choices of  $\alpha$ ,  $\beta$  and  $\gamma$  give a variety of power graphic contraction pairs of two mappings.

**Remarks 2.3** Let  $(X, p)$  be a complete partial metric space endowed with a directed graph  $G$ .

(R1) We may replace (1.1) with the following:

$$p^3(fx, gy) \leq \phi(p(x, y)p(x, fx)p(y, gy)) \quad (2.5)$$

to obtain conclusions of Theorem 2.1. Indeed, taking  $\alpha = \beta = \gamma = 1$  in Theorem 2.1, one obtains (2.5).

(R2) If we replace (1.1) by one of the following condition:

$$p^2(fx, gy) \leq \phi(p(x, y)p(x, fx)), \quad (2.6)$$

$$p^2(fx, gy) \leq \phi(p(x, y)p(y, gy)), \quad (2.7)$$

$$p^2(fx, gy) \leq \phi(p(x, fx)p(y, gy)), \quad (2.8)$$

then the conclusions obtained in Theorem 2.1 remain true. Note that

- (i) if we take  $\alpha = \beta = 1$  and  $\gamma = 0$  in (1.1), then we obtain (2.6),
- (ii) take  $\alpha = \gamma = 1$ ,  $\beta = 0$  in (1.1) to obtain (2.7),
- (iii) use  $\beta = \gamma = 1$ ,  $\alpha = 0$  in (1.1) and obtain (2.8).

(R3) Also, if we replace (1.1) by one of the following conditions:

$$p(fx, gy) \leq \phi(p(x, y)), \quad (2.9)$$

$$p(fx, gy) \leq \phi(p(x, fx)), \quad (2.10)$$

$$p(fx, gy) \leq \phi(p(y, gy)), \quad (2.11)$$

then the conclusions obtained in Theorem 2.1 remain true. Note that

- (iv) take  $\alpha = 1$  and  $\beta = \gamma = 0$  in (1.1) to obtain (2.9),
- (v) to obtain (2.10), take  $\beta = 1$ ,  $\alpha = \gamma = 0$  in (1.1),
- (vi) if one takes  $\gamma = 1$ ,  $\alpha = \beta = 0$  in (1.1), then we obtain (2.11).

**Remark 2.4** If we take  $f = g$  in a power graphic contraction pair, then we obtain fixed point results for a power graphic contraction.

#### Competing interests

The authors declare that they have no competing interests.

#### Authors' contributions

All authors read and approved the final manuscript.

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